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Original Article

On The Foundations of Fuzzy Number Theory and Fuzzy Diophantine Equations

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Abstract: *Unlike classical number theory, fuzzy logic has not yet been used to the development of a consistent number theory, despite the tremendous and quick advancements in its study and applications in many scientific domains. This study presents a conceptual understanding of fuzzy logic and fuzzy membership functions in number theory for the first time. It defines fuzzy congruence, the division process, and the greatest common divisor of integers with a fuzzy membership function. On the other hand, through numerous related theorems and accompanying illustrated examples, it offers many well-known Diophantine equations stated using fuzzy sets, as well as many features of fuzzy number theoretical systems. Furthermore, we anticipate that the study of fuzzy number theory will advance in the future, which is why we have raised a number of open research topics in this work.*

Keywords: *Fuzzy Number Theory; Fuzzy Diophantine Equation; Standard Fuzzy Number Theoretical System; Fuzzy Divisor; Fuzzy Prime; Fuzzy GCD.*

INTRODUCTION

Fuzzy sets have been distinguished throughout their history as a fertile field for finding applications in various branches of mathematics, practical and real life. Fuzzy theory has been widely used in decision-making applications, abstract algebra, in mathematical analysis, and also in engineering and computer science [21-50].

Although the study of fuzzy algebraic structures has diversified throughout itshistory through the study of rings, groups, vector spaces as well as matrices, it becomes clear that until now no consistent theory of fuzzy integer numbers has beenformulatedsimilar to classical number theory that isconcerned with theproperties of integers, as concepts such as fuzzy Diophantine equations, or fuzzy congruencies, or other central concepts in number theory havenotbeen formulated, due to the lack of conceptssuch asdivision, common divisors, primenumbers, and other basic concepts among fuzzy integer numbers.

We are attempting to develop an abstract concept of the fuzzy number theory system in this research because we are aware of this significant research gap. In this system, the concept of a fuzzy membership function can be used to define the division process, construct congruencies, and even Diophantine equations that are built on the set of integers with a fuzzy membership function. The primary goal of this book is to provide a gateway to the study of fuzzy number theory and to raise a multitude of open research problems that can further the field's understanding of both fuzzy and classical number theoretical systems.

In addition, this research work proposes some new models through which researchers interested in generalizing classical number theory can study the same familiar concepts from the perspective of a fuzzy membership function. This work is considered the first and foundational research aimed at opening a new research field, we call it fuzzy number theory.

General Versions of Number Theory

Due to the extensive nature of this field of theoretical mathematics, classical number theory has been generalized through numerous number theoretical systems that increase the set of integers. These systems include split-complex number theory [4,18], weak fuzzy number theory and its Diophantine equations [19–20], and lithogenic number theory [5–6,10,15–17]. Classical number theory is concerned with the study of the properties of integers and their applications in various fields. Our work on fuzzy number theory is an attempt to broaden the conventional understanding of integers and to pave the way for future study in this area. It is accompanied by a large number of outstanding problems that bear close resemblances to the issues faced by classical number theory.

1. Definition:

Let $\mathbb{Z}$ be the ring of integers, $\mu: \mathbb{Z} \to]0,1]$ be a membership function, we say that $(\mathbb{Z}, \mu)$ is a fuzzy number theoretical system.

Definition:

Consider the following membership function: $\mu: \mathbb{Z} \to]0,1]$ such that:

$$\mu(x) = \begin{cases} \frac{1}{|x|}; & x \neq 0 \\ 1; & x = 0 \end{cases}$$

Then $(\mathbb{Z}, \mu)$ is called the standard fuzzy number theoretical system (SFNTS)

For a fixed natural number $n \geq 2$, we define the n-standard fuzzy number theoretical system as $(\mathbb{Z}, \mu)$,with:

$$\text{1)} \quad \mu: \mathbb{Z} \to]0,1]; \mu(x) = \begin{cases} \frac{1}{|x|^n}; & x \neq 0 \\ 1; & x = 0 \end{cases}$$

Definition:

Let $(\mathbb{Z}, \mu)$ be a fuzzy number theoretical system (FNTS), then for $a, b \in \mathbb{Z}$, we say that $a|b$ if and only if $\frac{\mu(b)}{\mu(a)} \in \mathbb{Z}^+$.

Example:

Take $\mu: \mathbb{Z} \to]0,1]$ such that:

$\mu(1) = \mu(3) = \mu(5) = \frac{3}{4}$, $\mu(2) = \mu(4) = \frac{1}{4}$, $\mu(7) = \mu(6) = \frac{1}{3}$,

$\mu(x) = 1; x \geq 8$ or $x \leq 0$, then we have:

$6|7$, that is because $\frac{\mu(7)}{\mu(6)} = 1 \in \mathbb{Z}^+$,

$2|1$, that is because $\frac{\mu(1)}{\mu(2)} = 3 \in \mathbb{Z}^+$

Theorem:

Let $\mu: \mathbb{Z} \to]0,1]$, then:

1] If $a|b$, then $\mu(b) \geq \mu(a)$ $\forall\, a, b \in \mathbb{Z}$

2] $a|a$ for all $a \in \mathbb{Z}$.

3] If $a|b$ and $b|c$, then $a|c$ $\forall\, a, b, c \in \mathbb{Z}$. Proof:

1] $a|b$ if and only if $\frac{\mu(b)}{\mu(a)} \in \mathbb{Z}^+$, thus $\frac{\mu(b)}{\mu(a)} \geq 1$, so that $\mu(b) \geq \mu(a)$.

2] $\frac{\mu(a)}{\mu(a)} = 1 \in \mathbb{Z}^+$, hence $a|a$.

3] Assume that $a|b$ and $b|c$, we get:

$\frac{\mu(b)}{\mu(a)}, \frac{\mu(c)}{\mu(b)} \in \mathbb{Z}^+$, hence: $\frac{\mu(b)}{\mu(a)} \cdot \frac{\mu(c)}{\mu(b)} = \frac{\mu(c)}{\mu(a)} \in \mathbb{Z}^+$, thus $a|c$.

Definition:

Let $(\mathbb{Z}, \mu)$ be a (FNTS), and $a, b, c \in \mathbb{Z}$, we say that c = gcd(a, b) if and only if:

1) $c|a$ and $c|b$.
2) For any $t|a$ and $t|b$, then $t|c$; $t \in \mathbb{Z}$.

Example:

Define $\mu: \mathbb{Z} \to]0,1]$ such that:

$\mu(1) = \frac{1}{2}$, $\mu(2) = \frac{1}{4}$, $\mu(3) = \mu(4) = \frac{3}{4}$, $\mu(5) = \frac{1}{4}$, $\mu(x) = 1$ for all other $x \in \mathbb{Z}$.

We can see that: $2|3, 2|4, 5|3, 5|4, 5|2, 2|5$, hence $\gcd(3,4) \in \{2,5\}$.

Remark:

In fuzzy number theory, the (gcd) may not be unique in general.

Definition:

1] Let $a, b \in (\mathbb{Z}, \mu)$, we say (a,b) are relatively prime if and only if they not have a common divisor. 2] $a \in \mathbb{Z}$ is called a fuzzy prime element if and only if it has not any divisor different from itself.

Example:

Define $\mu: \mathbb{Z} \to]0,1]$ such that: $\mu(x) = 1; x \geq 4$ or $x \leq 0$ and: $\mu(1) = \frac{1}{3}, \mu(2) = \frac{1}{4}, \mu(3) = \frac{1}{5}$.

It is clear that: x is not a divisor of 1, 2, 3 for all different x in Z, hence 1, 2, 3 are fuzzy prime elements in $(\mathbb{Z}, \mu)$.

Remark:

In a (FNTS) $(\mathbb{Z}, \mu)$, if a, b are fuzzy prime elements, then (a, b) are relatively prime.

Remark:

If $a \in \mathbb{Z}$ is a fuzzy prime element, then $(a) \neq (b)$ for all $b \in \mathbb{Z}$ and $b \neq a$.

Theorem:

Let $(\mathbb{Z}, \mu)$ be a (FNTS), $a, b, c \in \mathbb{Z}$ such that:

$c|a$ and $c|b$, then there exists $r_1, r_2 \in \mathbb{Z}$ such that:

$r_1(a) + r_2\mu(b) = 0$. Proof:

$c|a$, $c|b$ implies that: $\frac{(a)}{\mu(c)} = \alpha \in \mathbb{Z}^+$, $\frac{(b)}{\mu(c)} = \beta \in \mathbb{Z}^+$, hence

$$\begin{cases} \mu(a) = \alpha\, \mu(c) \\ \mu(b) = \beta\, \mu(c) \end{cases}$$

Assume that $l = (\alpha, \beta)$, then $l = r1$, $l = r2\beta; r1, r1 \in \mathbb{Z}$, So that:

$$\begin{cases} r_1(a) = l\, \mu(c) \\ r_2(b) = l\, \mu(c) \end{cases}$$

Hence: $r1(a) + (-r2)\mu(b) = 0$

Remark:

If $c|a$ and $c|b$, then $\frac{(a)}{(b)} = \frac{r2}{r1} \in Q^+$

Also, $(c) = \frac{1}{\alpha+\beta}$

2. The classical formulas of some Diophantine equations in the standard system.

1] The linear Diophantine equation in two variables:

$$a.\, \mu(x) + b.\, \mu(y) = l.\, \mu(c) \Leftrightarrow \frac{a}{|x|} + \frac{b}{|y|} = \frac{l}{|c|} \Leftrightarrow a.\, |cy| + b.\, |cx| = l.\, |xy|;\ a, b, c, l, x, y \neq 0.$$

2] Fuzzy Pythagoras triples:

$$((x))^2 + (\mu(y))^2 = (\mu(z))^2 \Leftrightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2} \Leftrightarrow z^2(y^2 + x^2) = x^2y^2; \quad x, y, z \neq 0$$

3] Fuzzy Pythagoras quadruples:

$$((x))^2 + (\mu(y))^2 + (\mu(z))^2 = (\mu(t))^2 \Leftrightarrow \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{t^2}$$

4] Fuzzy Fermat's triples:

$$(\mu(x))^n + ((y)) = ((z)) \Leftrightarrow \frac{1}{|x|^n} + \frac{1}{|y|^n} = \frac{1}{|z|^n}; \quad n \geq 3$$

Example:

In the (SFNTS) $(\mathbb{Z}, \mu)$, consider the linear Diophantine equation in two variables.

$2(x) + 3(y) = (5)$, it is equivalent to:

2

$\frac{3}{|x|} + \frac{1}{|y|} = \frac{}{5} \Leftrightarrow 10|y| + 15|x| - |xy| = 0$

Put $Z = 10|y| - |xy|$, then:

$$\begin{cases} 15|x| + Z = 0 & (1) \\ Z = 10|y| - |x|.|y| & (2) \end{cases}$$

The equation (1) is solvable in $\mathbb{Z}$, let us consider the solution $(x_0, z_0) = (3, -45)$, By using (2), we get:

$-45 = 10|y| - 3|y| \Leftrightarrow |y| = \frac{-45}{7}$ which is contradiction.

In general, $Z = -15.|x|$, thus: $-15|x| = 10|y| - |x|.|y|$, so that $|y| = \frac{-15|x|}{10-|x|}$.

The possible solutions (x, y) should have the property $10 - |x| \mid -15|x|$, and $10 - |x| < 0$.

Now, we can write: $10 - |x| \mid -15|x|$, hence $\exists\, q \in \mathbb{Z}$ such that $-15|x| = (10 - |x|)$, thus $|x| = \frac{10q}{-15+q}$.

$-15\left(\frac{10q}{\quad}\right) > 0.$

Also, $|y| = \frac{\quad}{-15+}$ under the condition $-15 + q \mid 10q$ for all $q \in \mathbb{Z}$, and

$\frac{10q}{10-\frac{10q}{-15+q}}$ $\quad \frac{10q}{-15+q}$

For example: if q=20, then $|x| = 40$, $|y| = 20$, we get a solution.

3. Recommendations and open research problems:

This work can be considered as first work about fuzzy number theory, and the applications of fuzzy logic in Diophantine equations. We expect that this work will open a huge number of research problems that concern fuzzy number theory and Diophantine equations.

We list some of open problems.

1] Define a suitable fuzzy number theoretical system with some conditions on μ, and try to find the related formulas for congruencies, solutions for fuzzy linear Diophantine equations, fuzzy Pythagoras triples, and so on. 2] Find an algorithm to solve linear fuzzy Diophantine equation in two variables in the standard system or n- standard system. (or in any suitable system you defined).

3] Try to generalize famous Diophantine equations and number theoretical concepts in to the standard system/ n- standard system.

4] Try to use fuzzy number theory in cryptography, especially crypto-algorithms (RSA, ElGamal,…).

5] Try to find algorithms for generating Pythagoras triples, quadruples, and Fermat's triples in the standard system, n- standard system, or any other suitable fuzzy number theoretical system you defined.

$(\mu(a) + \mu(b)); \alpha, \beta \in \mathbb{Z}^+$.

Remark:

According to the previous theorem, we can see that $(\equiv)$ is an equivalence relation on the fuzzy number theoretical system $(\mathbb{Z}, \mu)$.

We denote the equivalence class of $(a) \in \mathbb{Z}$ modulo (c) as follows: $[a] = \{b \in \mathbb{Z}; b \equiv (mod c)\}$

Theorem:

Let $(\mathbb{Z}, \mu)$ be a (FNTS), then:

1] If $(a) = (b)$, then $a \equiv b \ (mod c)$ for all $c \in \mathbb{Z}$.

2] If $c|a$, $c|b$, then $a \equiv b \ (mod c)$.

3] If $c|a$, and $a \equiv b \ (mod c)$, then $c|b$. Proof:

1] $\frac{|\mu(a)-\mu(b)|}{\mu(c)} = 0 \in \mathbb{Z}^+$, hence $a \equiv b \ (mod c)$.

2] Assume that $c|a$, $c|b$, hence $\frac{\mu(a)}{\mu(c)}, \frac{\mu(b)}{\mu(c)} \in \mathbb{Z}^+$.

If $\mu(a) \geq \mu(b)$, then $\frac{|\mu(a)-\mu(b)|}{\mu(c)} = \frac{\mu(a)}{\mu(c)} - \frac{\mu(b)}{\mu(c)} \in \mathbb{Z}^+$

If $\mu(a) \leq \mu(b)$, then $\frac{|\mu(a)-\mu(b)|}{\mu(c)} = \frac{\mu(b)}{\mu(c)} - \frac{\mu(a)}{\mu(c)} \in \mathbb{Z}^+$,

Thus $a \equiv b \ (mod c)$.

3] Assume that $a \equiv b \ (mod c)$, $c|a$, then:

$\frac{\mu(a)}{\mu(c)}, \frac{|\mu(a)-\mu(b)|}{\mu(c)} \in \mathbb{Z}^+$, so that:

$\frac{\mu(a)}{\mu(c)} - \frac{\mu(b)}{\mu(c)} = \alpha \in \mathbb{Z}$, hence $\frac{\mu(b)}{\mu(c)} = -\alpha + \frac{\mu(a)}{\mu(c)} \in \mathbb{Z}^+$, which means that $c|b$.

Definition:

Let $(\mathbb{Z}, \mu)$ be a (FNTS), add $a, c \in \mathbb{Z}$, then:

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